Week 3: Basic regression

3. Linear model interpretation

Stat 140 - 04

Mount Holyoke College

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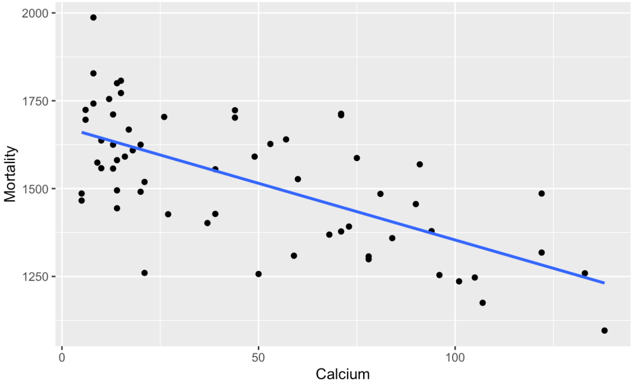
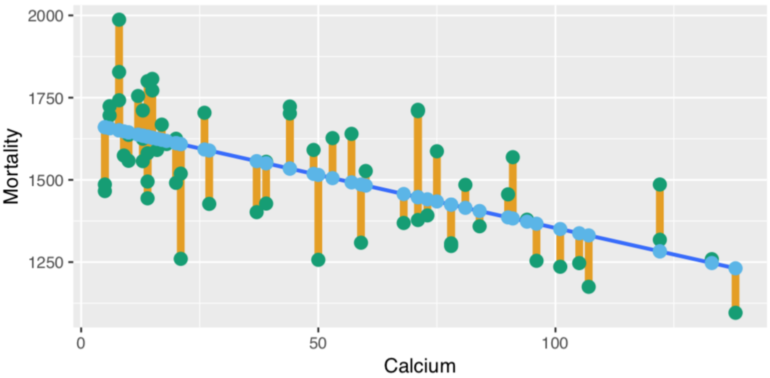
Slides posted at *http://sshanshans.github.io/stat140*

1. Yesterday: line of best fit
2. Main ideas
    1. Find the least square line by hand
    2. Interpret intercept and slope
3. Summary


2000
1750
1500
1250
Mortality
0
50
100
Calcium

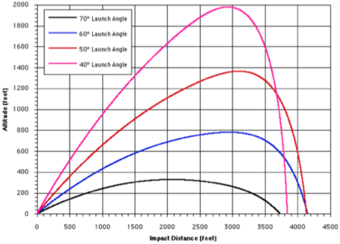
The line of best fit is the line for which the sum of the squared residuals is smallest, the **least squares line**.



The algebraic equation for a line is

$$Y = b_0 + b_1 X$$

The use of coordinate axes to show functional relationships was invented by Rene Descartes (1596-1650). He was an artillery officer, and probably got the idea from pictures that showed the trajectories of cannonballs.



- x: the explanatory variable (calcium concentration)
- y: the response variable (mortality rate)
- $\bar{x}, \bar{y}$: sample mean of x and y
- s_x, s_y: sample standard deviation of x and y
- R: correlation between x and y

The least square line has

- slope:

$$b_1 = \frac{s_y}{s_x} R$$

- intercept (the value at $x = 0$):

$$b_0 = \bar{y} - b_1 \bar{x}$$

Suppose we have the following information.

	mortality rate (y)	calcium concentration (x)
mean	$\bar{y} = 1524$	$\bar{x} = 47$
sd	$s_y = 188$	$s_x = 38$
correlation		$R = -0.65$

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3. Write out the linear model.
$$\widehat{\text{Mortality}} = 1675 - 3 \text{ Calcium}$$

In general, the regression line is

$$\hat{y} = b_0 - b_1 x$$

1. **Slope** b_1: Slopes are always expressed in y-units per x-unit. They tell how the y-variable changes (in its units) for a one-unit change in the x-variable.
2. **Intercept** b_0: the value the line takes when x is zero

How to interpret intercept and slope in the context of data?

Units:

- x-variable: calcium concentration (parts per million)
- y-variable: mortality rate (deaths per 100,000 population)

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we can expect, on average, to have 3 units less in y-variable.

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Less formally, for each additional parts per million increase in calcium concentration, the predicted number of mortality rate decreases by 3 deaths per 100,000 population.

Algebraically, that's the value the line takes when x is zero.

Here, our model predicts that when the water does not have any calcium, on average, the mortality rate is 1676 deaths per 100,000 population.

Note that the intercept serves only as a starting value for our predictions, and we don't interpret it as a meaningful predicted value.

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